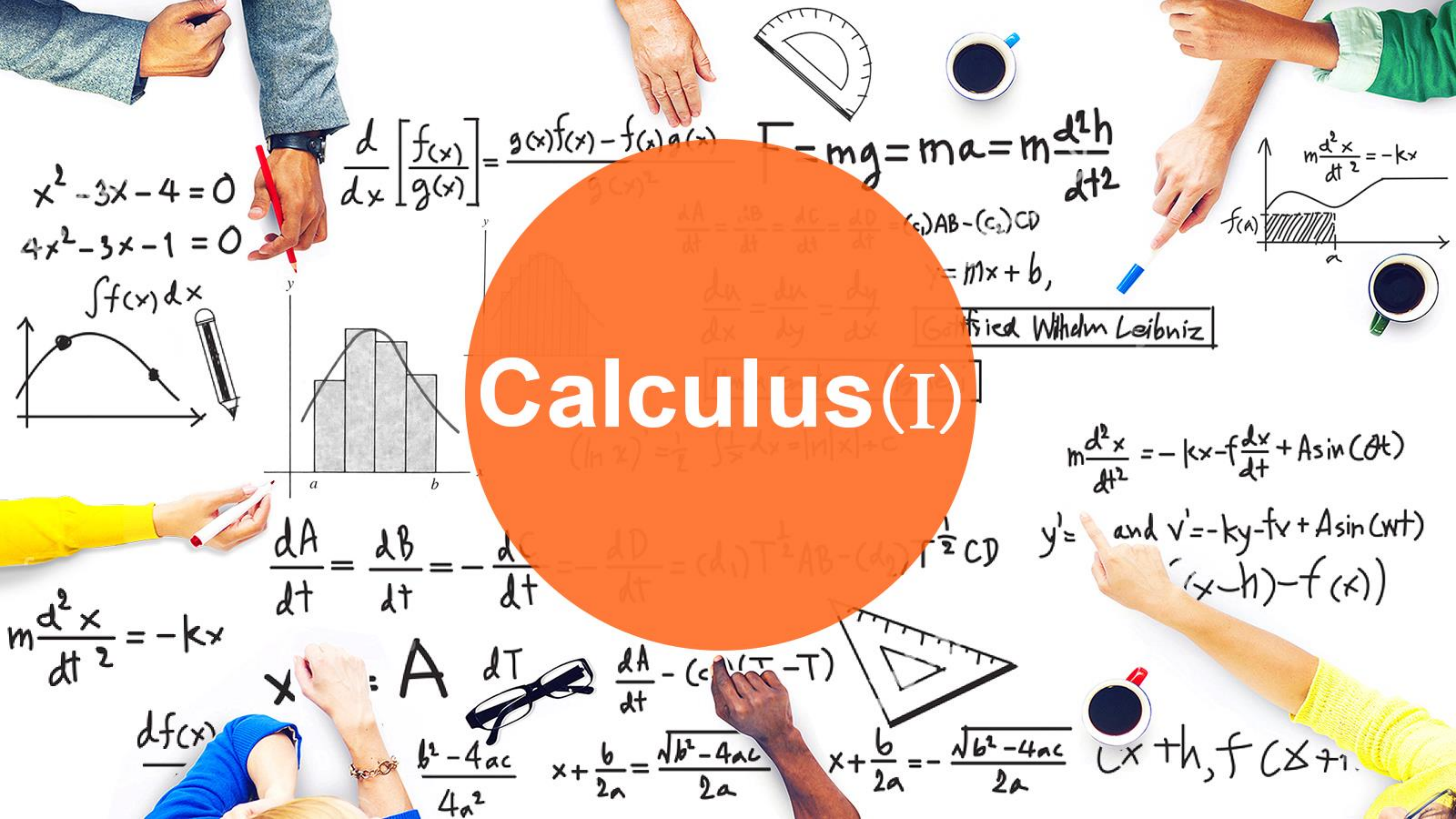
# Calculus(Ⅰ)



$x^2-3x-4=0$
$4x^2-3x-1=0$
$\int f(x)\,dx$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=$
$F=mg=ma=m\frac{d^2h}{dt^2}$
$m\frac{d^2x}{dt^2}=-kx$
$m\frac{d^2x}{dt^2}=-kx-f\frac{dx}{dt}+A\sin(\theta t)$
and $v'=-ky-fv+A\sin(wt)$
Gottfried Wilhelm Leibniz
$\frac{dA}{dt}=\frac{dB}{dt}=-\frac{dC}{dt}$
$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$
$x+\frac{b}{2a}=-\frac{\sqrt{b^2-4ac}}{2a}$
$\frac{b^2-4ac}{4a^2}$

# Integration by Parts

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$$\int \boxed{xe^x}\, dx = ? \qquad \int \boxed{x \ln x}\, dx = ? \qquad \int \boxed{\arcsin x}\, dx = ?$$

* **Characteristics:** The product of different functions.

* **Thinking idea:** The derivation rule of product.

# Integration by Parts

Let functions $u = u(x)$ and $v = v(x)$

has continuous derivatives.

$$(uv)' = u'v + uv' \Rightarrow \boxed{uv' = (uv)' - u'v}$$

**Integral: on both sides**

Then, we can obtain: $\int uv'dx = \int \boxed{(uv)'} dx - \int u'vdx$

$$\boxed{\int udv = uv - \int vdu \quad \text{Integration by parts}}$$

# Integration by Parts

$$\boxed{\int u\,dv} = uv - \boxed{\int v\,du}$$

**Integration by Parts**

The general principle of choosing $u$ and $dv$

(1) $v$ is easy to obtain;

(2) $\int v\,du$ is more easier than $\int u\,dv$ .

# Example 1

Find $\int x\cos x dx$  $\int u dv = uv - \int v du$

Method1 $u = \cos x,\ v = \frac{x^2}{2},\ \Rightarrow du = -\sin x dx,\ dv = x dx,$

Then, $\int x\cos x dx = \int \cos x d(\frac{x^2}{2})$

$$= \frac{x^2}{2}\cos x + \boxed{\int \frac{x^2}{2}\sin x dx}$$

Obviously, choosing $u, dv$ is very important,

Otherwise, the integration becomes more difficult.

# Example 1

Find $\int x\cos x dx$ $\int u dv = uv - \int v du$

Method2 $u = x,$ $v = \sin x$ ➡ $du = dx,\ \ dv = \cos x dx,$

$$\int x\cos x dx = \int x d\sin x$$

$$= x\sin x - \int \sin x dx$$

$$= x\sin x + \cos x + C$$

# Example 2

Find $\int xe^x dx$ $\int udv = uv - \int vdu$

Let $\boxed{u = x, \quad v = e^x}$ $du = dx, dv = e^x dx$

$$\int xe^x dx = \int xde^x$$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

# Example 3

Find $\int x^2 e^x dx$ $\int u dv = uv - \int v du$

Let $\boxed{u = x^2, \quad v = e^x}$ $du = 2x dx, \quad dv = e^x dx$

$$\int x^2 e^x dx = \int x^2 de^x$$

$$= x^2 e^x - 2\int x e^x dx$$

**(by parts again)**

$$= x^2 e^x - 2(x e^x - e^x) + C$$

# Example 4

Find $\int x^3 \ln x dx$ $\int udv = uv - \int vdu$

$$\boxed{u = \ln x, \;\; v = \frac{x^4}{4}, du = \frac{1}{x}dx, \;\; dv = x^3 dx}$$

$$\int x^3 \ln x dx = \int \ln x d\frac{x^4}{4} = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^4 d\ln x$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$

# Example 5

Find $\int e^x \sin x dx$ $\boldsymbol{u}$

$$= \int \underbrace{\sin x}_{u} \underbrace{de^x}_{dv} = e^x \sin x - \int e^x d(\sin x)$$

$$= e^x \sin x - \int e^x \underbrace{\cos x}_{u} dx = e^x \sin x - \int \underbrace{\cos x}_{u} \underbrace{de^x}_{dv}$$

$$= e^x \sin x - (e^x \cos x - \int e^x d \cos x)$$

$$= e^x (\sin x - \cos x) - \boxed{\int e^x \sin x dx}$$

Pay attention to the circular form

$$\therefore \int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

# Summary（Ⅰ）

Integration by Parts: $\int u\,dv = uv - \int v\,du$

$$\int \underbrace{\boxed{P_n(x)}}_{u} \sin ax\,dx,\quad \int \underbrace{\boxed{P_n(x)}}_{u} \cos ax\,dx,\quad \int \underbrace{\boxed{P_n(x)}}_{u} e^{kx}\,dx,$$

$k, a$ are constants, $P_n(x)$ is an $n$-order polynomial.

# Summary (II)

$$\int P_n(x)\underbrace{\boxed{\arcsin x}}_{u}\,dx,\quad \int P_n(x)\underbrace{\boxed{\arctan x}}_{u}\,dx,$$

$$\int P_n(x)\underbrace{\boxed{\ln x}}_{u}\,dx$$

We know, using the following two formulas

$$\arcsin x+\arccos x=\frac{\pi}{2},\qquad \arctan x+\operatorname{arccot} x=\frac{\pi}{2},$$

These integrations $\int P_n(x)\arccos x\,dx,\ \int P_n(x)\operatorname{arccot} x\,dx$

can be transformed into: $\int P_n(x)\arcsin x\,dx,\ \int P_n(x)\arctan x\,dx$

# Summary (III)

$$\int e^{kx}\sin(ax+b)dx, \int e^{kx}\cos(ax+b)dx,$$

$k, a$ are constant numbers.

We can select ***u*** and ***dv*** by random

***u*** should be the same type function every time.

# Questions and Answers

$$\int udv = uv - \int vdu$$

$$\int \arcsin x dx$$

$$= x\arcsin x - \int x d \arcsin x$$

$$= x\arcsin x - \int x \frac{1}{\sqrt{1-x^2}} dx$$

$$= x\arcsin x + \sqrt{1-x^2} + C$$

# Questions and Answers

$$\int u\,dv = uv - \int v\,du$$

$$\int_1^2 t^6 \ln t\,dt$$

$$= \frac{1}{7}\int_1^2 \ln t\,dt^7$$

$$= \frac{1}{7}(t^7 \ln t)\Big|_1^2 - \frac{1}{7}\int_1^2 t^7 \cdot \frac{1}{t}\,dt$$

$$= \frac{1}{7}(2^7 \ln 2) - \frac{1}{7}\cdot\frac{t^7}{7}\Big|_1^2$$

$$= \frac{128}{7}\ln 2 - \frac{127}{49}$$

# Questions and Answers

$$\boxed{\int e^x \cos x dx}$$

$$= \int \cos x de^x \quad = e^x \cos x - \int e^x d(\cos x)$$

$$= e^x \cos x + \int \sin x de^x$$

$$= e^x \cos x + (\sin x e^x - \int e^x d(\sin x))$$

$$= e^x(\sin x + \cos x) - \boxed{\int e^x \cos x dx}$$

$$\therefore \int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + C$$

# Integration by Parts